Further Statistics – CRV Introduction

Patrons are reminded that for $f(x)$ to be a Probability Density Function (pdf)

$$f(x) \geqslant 0 \text{ for all } x \qquad \text{and} \qquad \int_{-\infty}^{\infty} f(x)\,dx = 1.$$

Probabilities are represented by areas, so

$$P(a \leqslant X \leqslant b) = \int_a^b f(x)\,dx.$$

1. Given the pdf

$$f(x) = \begin{cases} kx & 0 \leqslant x \leqslant 3 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Sketch the pdf.
   (b) Find the value of k. $\boxed{k = \frac{2}{9}}$
   (c) Find $P(X = 2)$. $\boxed{0}$
   (d) Find $P(2 \leqslant X \leqslant 3)$. $\boxed{\frac{5}{9}}$

2. Given the pdf

$$f(x) = \begin{cases} kx & 0 \leqslant x \leqslant 1 \\ k & 1 < x \leqslant 3 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Sketch the pdf.
   (b) Find the value of k. $\boxed{k = \frac{2}{5}}$
   (c) Find $P\left(\frac{1}{2} \leqslant X \leqslant 2\right)$. $\boxed{\frac{11}{20}}$

3. Given the pdf

$$f(x) = \begin{cases} kx^2 & -1 \leqslant x \leqslant 3 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Sketch the pdf.
   (b) Find the value of k. $\boxed{k = \frac{3}{28}}$
   (c) Find $P(0 \leqslant X \leqslant 2)$. $\boxed{\frac{2}{7}}$

4. Given the pdf

$$f(x) = \begin{cases} \frac{k}{x} & 1 \leqslant x \leqslant 4 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Sketch the pdf.
   (b) Find the value of k. $\boxed{k = \frac{1}{\ln 4} = \frac{1}{2\ln 2}}$
   (c) Find $P(1 \leqslant X \leqslant 2)$. $\boxed{\frac{1}{2}}$

5. Given the pdf

$$f(x) = \begin{cases} kx^2 & 0 \leqslant x \leqslant 1 \\ -kx & -1 \leqslant x < 0 \\ 0 & \text{otherwise} \end{cases}$$

(a) Sketch the pdf.

(b) Find the value of k. $\boxed{k = \frac{6}{5}}$

(c) Find $P\left(-\frac{1}{2} \leqslant X \leqslant \frac{1}{2}\right)$. $\boxed{\frac{1}{5}}$

6. Given the pdf

$$f(x) = \begin{cases} ke^{-x} & x \geqslant 0 \\ 0 & \text{otherwise} \end{cases}$$

(a) Sketch the pdf.

(b) Find the value of k. $\boxed{k = 1}$

(c) Find $P(1 \leqslant X \leqslant 2)$. $\boxed{\frac{e-1}{e^2}}$